TRINITY COLLEGE

# Year 12 Mathematics Application
# Test 2 2017

Section 1 Calculator Free

**Sequences and Networks**

**STUDENT'S NAME** Solutions

**DATE:** Thursday 30th March **TIME:** 15 minutes **MARKS:** 15

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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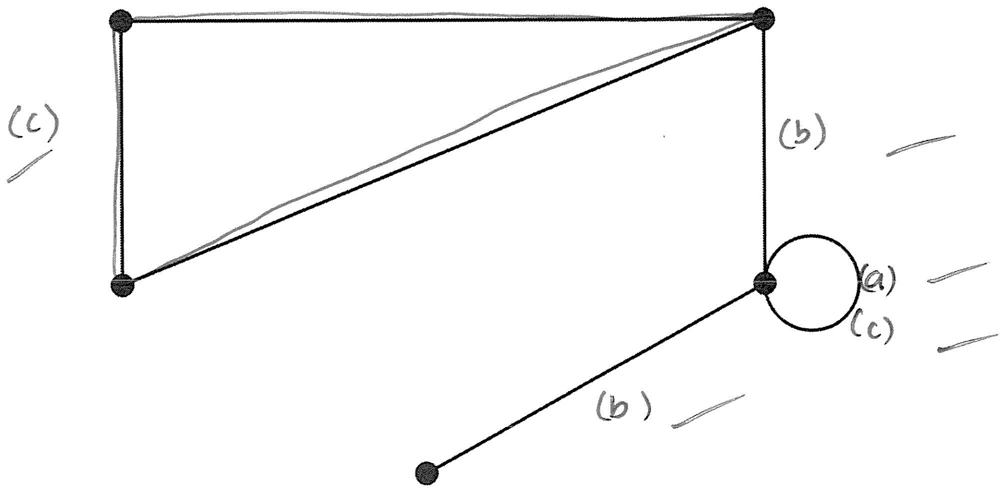
1. (5 marks)




Identify and label each of the following components of the above network.

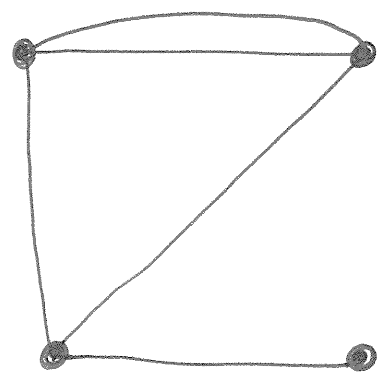
(a) any loop(s) [1]

(b) any bridge(s) [2]

(c) any cycle(s) [2]

2. (3 marks)

Draw a connected planar graph such that it satisfies the following criteria. The network must have 4 vertices, 5 edges (including one bridge) and 3 faces.



4 V ✓

5E (inc. bridge) ✓

3F ✓

3. (4 marks)

Given the arithmetic sequence 4, 1, -2, -5, -8…

(a) Identify the first term. [1]

4 ✓

(b) Identify the common difference [1]

$-3$ ✓

(c) State the simplified <u>general</u> rule [2]

$T_n = 4 + (n-1)(-3)$ ✓

$= 4 - 3n + 3$

$= 7 - 3n$ ✓

4. (3 marks)

A geometric progression has a third term of 6 and fifth term of 54. Determine the:

(a) Common ratio [2]

$\frac{54}{6} = r^2$ ✓

$9 = r^2$

$r = 3$ ✓

(b) First term [1]

$T_1 = 2/3$ ✓

TRINITY COLLEGE

# Year 12 Mathematics Applications
# Test 2 2017

## Section 2 Calculator Assumed
**Sequences and Networks**

**STUDENT'S NAME** ______________________________

**DATE**: Thursday 30th March **TIME:** 35 minutes **MARKS**: 35

**INSTRUCTIONS:**

| | |
|---|---|
| Standard Items: | Pens, pencils, drawing templates, eraser |
| Special Items: | Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment) |

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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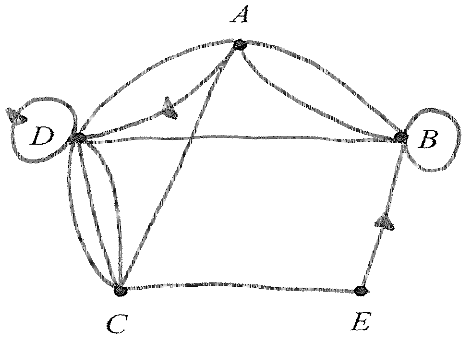
5. (4 marks)

$$\text{From:}\quad \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \text{To:} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & \cancel{2} & \cancel{1} & \cancel{2} & 0 \\ \cancel{2} & \cancel{2} & 0 & \cancel{1} & 0 \\ \cancel{1} & 0 & 0 & \cancel{3} & \cancel{1} \\ \cancel{1} & \cancel{1} & \cancel{3} & \cancel{1} & 0 \\ 0 & \cancel{1} & \cancel{1} & 0 & 0 \end{bmatrix} \end{array}$$

Draw the directed network corresponding to the above adjacency matrix




– 1 per incorrect

6. (7 marks)

A runner is attempting to complete a 24-hour race to raise money for charity. In the first hour, the runner travels 6 km, in the second hour travels 5.7 km and in the third hour 5.415 km.

(a) Show that there is a geometric relationship between the distances run and that the common ratio is 0.95. [2]

$$\frac{5.7}{6} = \frac{5.415}{5.7} = 0.95$$

(b) Write a <u>recursive</u> rule for the distance travelled each hour. [2]

$$T_n = 0.95T_{n-1}, \quad T_1 = 6$$

(c) How far, to the nearest metre, will the runner travel during the $10^{th}$ hour? [1]

$$T_{10} = 3.781 \text{ km}$$

(d) How far will the runner travel in the last 3 hours? [2]

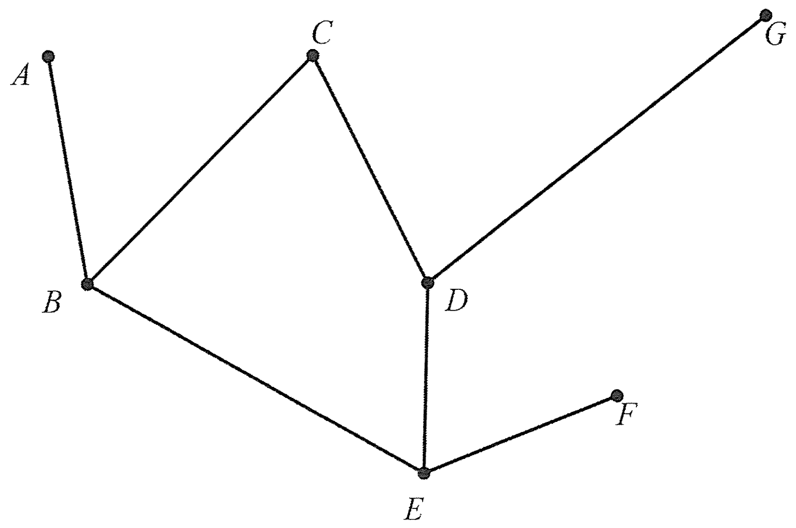
$$T_{22} = 2.043$$

$$T_{23} = 1.941$$

$$T_{24} = 1.844$$

$$T_{22-24} = 5.828 \text{ km}$$

7. (6 marks)

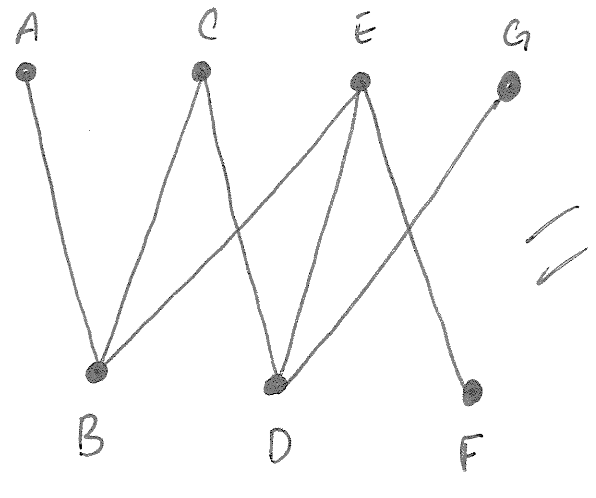



(a) Given the above network determine the degree of each of the vertices below.

(i) A 1 [1]

(ii) D 3 [1]

(b) Redraw the network so that it is obviously bipartite and hence state the two groups of vertices [4]



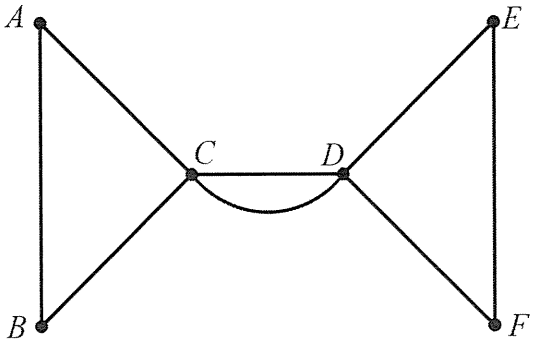

$\{A, C, E, G\}$.

$\{B, D, F\}$.

8. (6 marks)

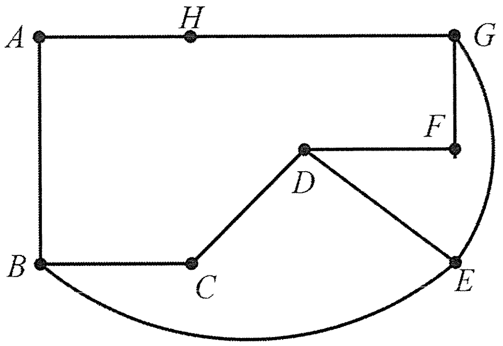
Classify the following networks as Eulerian, semi-Eulerian, Hamiltonian, semi-Hamiltonian or none of the previous terms.

(a) [2]




Eulerian ✓
semi-Hamiltonian ✓

(b) [2]

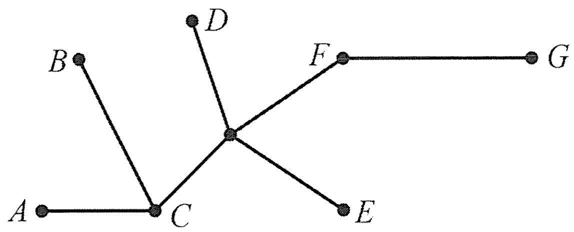



semi-Hamiltonian

✓✓

−1 for any extra

(c) [2]




neither

✓✓

−1 for any extra

9. (5 marks)

A crayfish farm increases its population by a constant percentage every month before a constant amount of crayfish are taken out to be sold. The first order linear recurrence relation for this is:

$$C_n = 1.15C_{n-1} - 70, \; C_0 = 750$$

(a) By what percentage does the population increase each month? [1]

15%

(b) How many crayfish will be in the farm after 7 months? [1]

$T_7 = 1220.33$

$\therefore 1220$ crayfish

(c) The amount of crayfish at 7 months is close to capacity for the farm and it is decided that the population should be stabilised. How many crayfish must be taken out every month to maintain a stable population of that given in part (b)? [1]

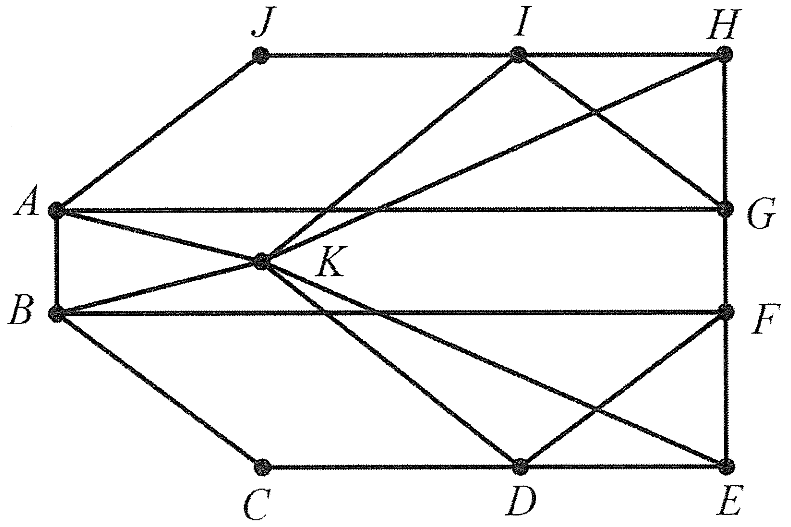
$0.15 \times 1220$

$= 183$ crayfish

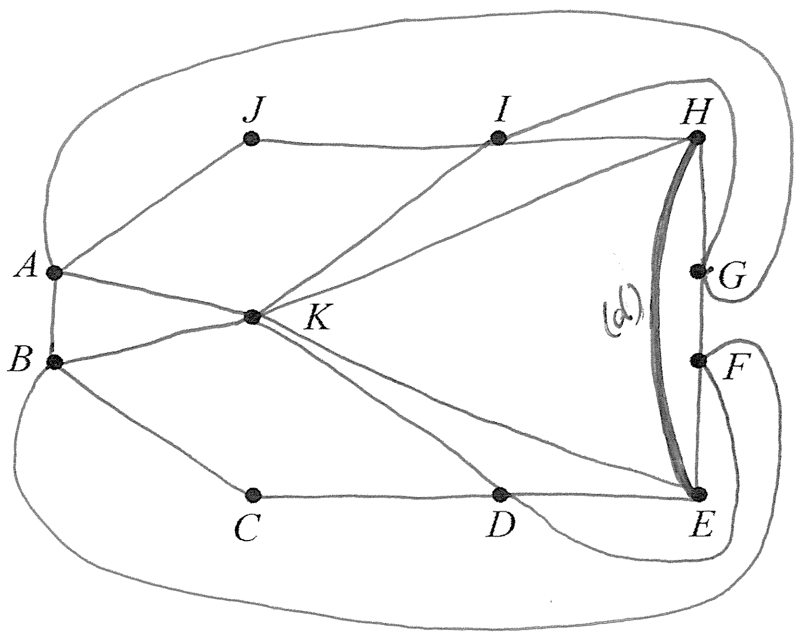
(d) State the new first order linear recurrence relation, where $C_0$ is the population at 7 months. [2]

$C_n = 1.15C_{n-1} - 183, \quad C_0 = 1220$

10. (7 marks)




(a) Redraw the network above so that it is planar. [2]




(b) Use Euler's formula to verify that part (a) is correct. [1]

v =
e =
f =

$11 + 11 = 20 + 2$

$22 = 22$

$11 + 11 - 20 = 2$

$2 = 2.$

(c) Given that a Hamiltonian cycle is possible state a path that satisfies this. [2]

AKBCDEFGHIJA

✓ Hamiltonian

✓ cycle.

(d) Is it possible to add a single edge to this network to make it Eulerian whilst continuing to be planar? If so add it to your network in part (a). [2]

yes. ✓

EH ✓